University of California, Berkeley
Physics 105 Fall 2000 Section 2 (*Strovink*)

EXAMINATION 3

Directions. Do both problems (weights are indicated). This is a closed-book closed-note exam except for three $8\frac{1}{2} \times 11$ inch sheets containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he will not give hints and will be obliged to write your question and its answer on the board. Roots, circular functions, *etc.*, may be left unevaluated if you do not know them. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

1. (50 points)
A satellite is in elliptical orbit about the earth (neglect any effects of the moon or sun). Its radius r is proportional to

$$r \propto \frac{1}{1+\epsilon\cos\theta} ,$$

where θ is the azimuthal angle of the orbit, and ϵ is the ellipse's *eccentricity*. For simplicity take $r_{\text{max}} = 3r_{\text{min}}$, so that $\epsilon = \frac{1}{2}$.
(**a**) (10 points)
Using any relevant theorem(s), write down the ratio $-\langle T\rangle/\langle U\rangle$, where T and U are the satellite's kinetic and potential energies, and $\langle\rangle$ is the time average over one full orbit.
(**b**) (20 points)
When $r = r_{\text{max}}$, what is $-T/U$? [*Hint*: the satellite's total energy is inversely proportional to the semimajor axis of its orbit. If you don't remember the constant of proportionality, you can deduce it by considering the special case of a circular orbit.]
(**c**) (20 points)
When $r = r_{\text{max}}$, a rocket on board the satellite fires a very brief burst, consuming fuel of negligible mass. Immediately after the burst, the satellite's total energy (normalized to zero at $r = \infty$) changes by a factor C, but its direction of motion remains the same; the satellite's orbit becomes *circular*. Solve for C.

2. (50 points)
When undriven, an undamped oscillator (*i.e.* a mass on a spring) satisfies the equation

$$\ddot{x} + \omega_0^2 x = 0 ,$$

where ω_0 is a positive constant. For $t < 0$ it is at rest at the origin: $x(t<0) = 0$.
(**a**) (20 points)
For this part, suppose that the mass is given a *quick tap* at $t = 0$, *i.e.*

$$\begin{aligned} x(t=0^+) &= 0 \\ \dot{x}(t=0^+) &= v_0 , \end{aligned}$$

where v_0 is a positive constant. Solve for $x(t)$ for $t > 0$. [*Hint*: your solution should be equivalent to $v_0\,G(t)$, where $G(t)$ is the *Green function* for this oscillator.]
(**b**) (30 points)
For this part, suppose instead that the mass is given a *steady push* that begins at $t = 0$ and lasts for one period. That is, suppose that the force F on the mass, divided by the mass m, is such that $F/m = a(t)$, where

$$\begin{aligned} a(t) &= a_0 \quad (0 < t < \tfrac{2\pi}{\omega_0}) \\ &= 0 \quad\quad \text{otherwise} , \end{aligned}$$

where a_0 is a positive constant. Solve for $x(t)$ after the push is finished, *i.e.* for $t > 2\pi/\omega_0$.